Complex Number

$z = x + iy$

x: Real part

y: Imaginary part.

Real Number is a Subset of Complex Number.

for ex: $z = -2 + 3i$

Real part of $z = Re(z) = -2$

Imaginary part of $z = Im(z) = 3$.

(i) Algebra:

(i) Addition: Given: $z_1 = 2+i$ $z_2 = -3-7i$

$z_1 + z_2 = 2+i - 3 - 7i$

$= 2 - 3 + i - 7i$

$= -1 + -6i$

$= -1 - 6i$

(ii) Subtraction: $z_1 - z_2 = (2+i) - (-3-7i)$

$= 2 + i + 3 + 7i = 5 + 8i$

(i) Real से Real

And

Imaginary से

Imaginary.

3) Power of i :-

$i \rightarrow$ iota

$i = \sqrt{-1}$

$(i)^2 = (\sqrt{-1})^2$

$i^2 = -1$

$i^3 = i^{2+1} = i^2 \cdot i$

$= -i$

$i^4 = i^2 \cdot i^2$

$= -1 \times -1$

$= 1$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = \underline{1}$

Reference

$(i)^{315} = ?$

$= (i)^{4\times 78 + 3}$

$= (i^4)^{78} \cdot i^3$

$= (1)^{78} \cdot (-i)$

$= 1 \times -i$

$= -i$

$$\begin{array}{r} 78 \\ 4\overline{)315} \\ 28 \\ \hline 35 \\ 32 \\ \hline 3 \end{array}$$

$(i)^{1479} = (i)^3 = -i$

$$\begin{array}{r} 369 \\ 4\overline{)1479} \\ 12 \\ \hline 27 \\ 24 \\ \hline 39 \\ 36 \\ \hline (3) \end{array}$$

Properties: (i) $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$ (Sum of four consecutive power of i is zero).

for ex: $i + i^2 + i^3 + i^4$

$\cancel{i} - \cancel{1} - \cancel{i} + \cancel{1} = 0$

(ii) $(i^n)(i^{n+1})(i^{n+2})(i^{n+3}) = -1$ (Product of four consecutive power of $i = -1$).

for ex: $i \times i^2 \times i^3 \times i^4 = i \times -1 \times -i \times 1 = +i^2 = -1$

(4) Multiplication:- for ex: $z_1 = 2+3i$ | $z_2 = -1+2i$

$z_1 z_2 = (2+3i)(-1+2i)$

$= -2+4i-3i+6(i^2)$

$= -2+i-6$

$= -8+i$

$z = x+iy$

Standard form of Complex No.

(5) Division:- for ex: $z_1 = 1+i$ $z_2 = 1-i$

$\frac{z_1}{z_2} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{1+i^2+2i}{1-i^2} = \frac{1-\cancel{1}+2i}{2} = \frac{\cancel{2}i}{\cancel{2}} = i$

$i^2 = -1$

Note: (i) $(1+i)^2 = 2i$ | $(1-i)^2 = -2i$

(ii) $\left(\frac{1+i}{1-i}\right) = i$ | $\boxed{\frac{1-i}{1+i} = \frac{1}{i} \times \frac{i^4}{} = i^3 = -i}$

Qus: find $\left(\frac{1+i}{1-i}\right)^{2000}$

$= (i)^{2000} = \left((i)^4\right)^{500} = (1)^{500} = 1$ Ans

(6) Inverse :- (i) Additive Inverse :

$\therefore$ Additive Inverse of $(z) = -z$

Given :- $z = 1+i$; find Additive inverse ?

$\Rightarrow -z = \boxed{-1-i}$

(ii) Multiplicative Inverse (M.I) :- M.I of z is $= \frac{1}{z}$

Given :- (i) $z = 1+i$ find M.I $(z) = ?$

$M.I(z) = \frac{1}{z} = \frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{1-i^2} = \frac{1-i}{2}$ Ans

Note: M.I: $\dfrac{\bar{z}}{|z|^2}$

Important Points: (i) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ is Not Valid for a is $-ve$ & b is $-ve$.

(ii) Inequality has No meaning in Complex No.

(a) $z_1 = -2+3i$ $\quad z_2 = 1+7i$

$z_1 > z_2$ OR $z_2 > z_1$

Until Imaginary part is zero.

$\therefore$ $z_1 = a + ib$ $\quad$ $z_2 = c + id$

$z_1 > z_2$ OR $z_2 > z_1$ is Valid when $Im(z_1) = Im(z_2) = 0$

(b) To Compare, two Complex Number, we can use Modulus also. $|z_1| = \sqrt{a^2+b^2}$; $|z_2| = \sqrt{c^2+d^2}$

Now, we can compare.

Conjugate of Complex Number :-

$z = x + iy$

$\bar{z} = \overline{x + iy}$

$\bar{z} = x - iy$

Ex :- $z = 1 + i$

$\bar{z} = 1 - i$

$z = 2i$

$\bar{z} = \overline{2i}$

$\bar{z} = -2i$

$z = 3$

$\bar{z} = \bar{3}$

$\bar{z} = 3$

Properties :-

(i) $\overline{(\bar{z})} = z$

(ii) $\overline{(z_1 z_2)} = \bar{z}_1 \bar{z}_2$ (in general, $\overline{(z_1 z_2 z_3 \ldots z_n)} = \bar{z}_1 \bar{z}_2 \bar{z}_3 \ldots \bar{z}_n$

(iii) $\overline{(z_1 \pm z_2)} = \bar{z}_1 \pm \bar{z}_2$

(iv) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

(v) $\overline{(z^n)} = (\overline{z})^n$

(vi) $z = x + iy$

let $y = 0$

$z = x$

Purely Real Complex No.

$\overline{z} = \overline{x}$

$\overline{z} = x$

$\overline{z} = z$

let $x = 0$

$z = iy$

Purely Imaginary Complex No.

$\overline{z} = \overline{iy}$

$\overline{z} = -iy$

$\overline{z} = -z$

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$z = x + iy$

$\bar{z} = x - iy$

$z + \bar{z} = x + \cancel{iy} + x - \cancel{iy}$

$z + \bar{z} = 2x$

$$\boxed{z + \bar{z} = 2\,Re(z)}$$

$z - \bar{z} = \cancel{x} + iy - \cancel{x} + iy$

$= 2iy$

$$\boxed{z - \bar{z} = 2i\,Im(z)}$$

⇒ Modulus of Complex Number :-

$z = x + iy.$

$|z| = \sqrt{x^2 + y^2}$

$= \sqrt{(Re(z))^2 + (Im(z))^2}$

$= +ve.$

Properties :-

(i) $z = x + iy$

$\bar{z} = x - iy$

$|z| = |\bar{z}|$ ✓

(ii) $|z_1 z_2| = |z_1||z_2|$

(iii) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

(iv) $|z^n| = |z|^n$

(5) $|z_1| - |z_2| \le |z_1 + z_2| \le |z_1| + |z_2|$

also

$|z_1| - |z_2| \le |z_1 - z_2| \le |z_1| + |z_2|$

Hint: To find maxm & minimum value of $|z|$.

(6) (i) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

(ii) $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

Where θ_1 = argument of z_1 & $\theta_2 = \arg(z_2)$
$= \arg(z_1)$.

(7) $z = x + iy \mid \bar{z} = x - iy$.

$z\bar{z} = (x+iy)(x-iy)$

$= x^2 - (iy)^2 = x^2 + y^2$

$z\bar{z} = |z|^2$

Jaruba

Ques: (i) $z = \dfrac{1+i\sin\theta}{1-i\sin\theta}$ find $|z| = ?$

NDA 2020 (2)

$\dfrac{1+i\sin\theta}{1-i\sin\theta} \times \dfrac{1+i\sin\theta}{1+i\sin\theta} \Rightarrow (?)$

$|z| = \left|\dfrac{1+i\sin\theta}{1-i\sin\theta}\right|$

$= \dfrac{|1+i\sin\theta|}{|1-i\sin\theta|}$

$|z| = 1$

Geometry of Conjugate & Modulus!

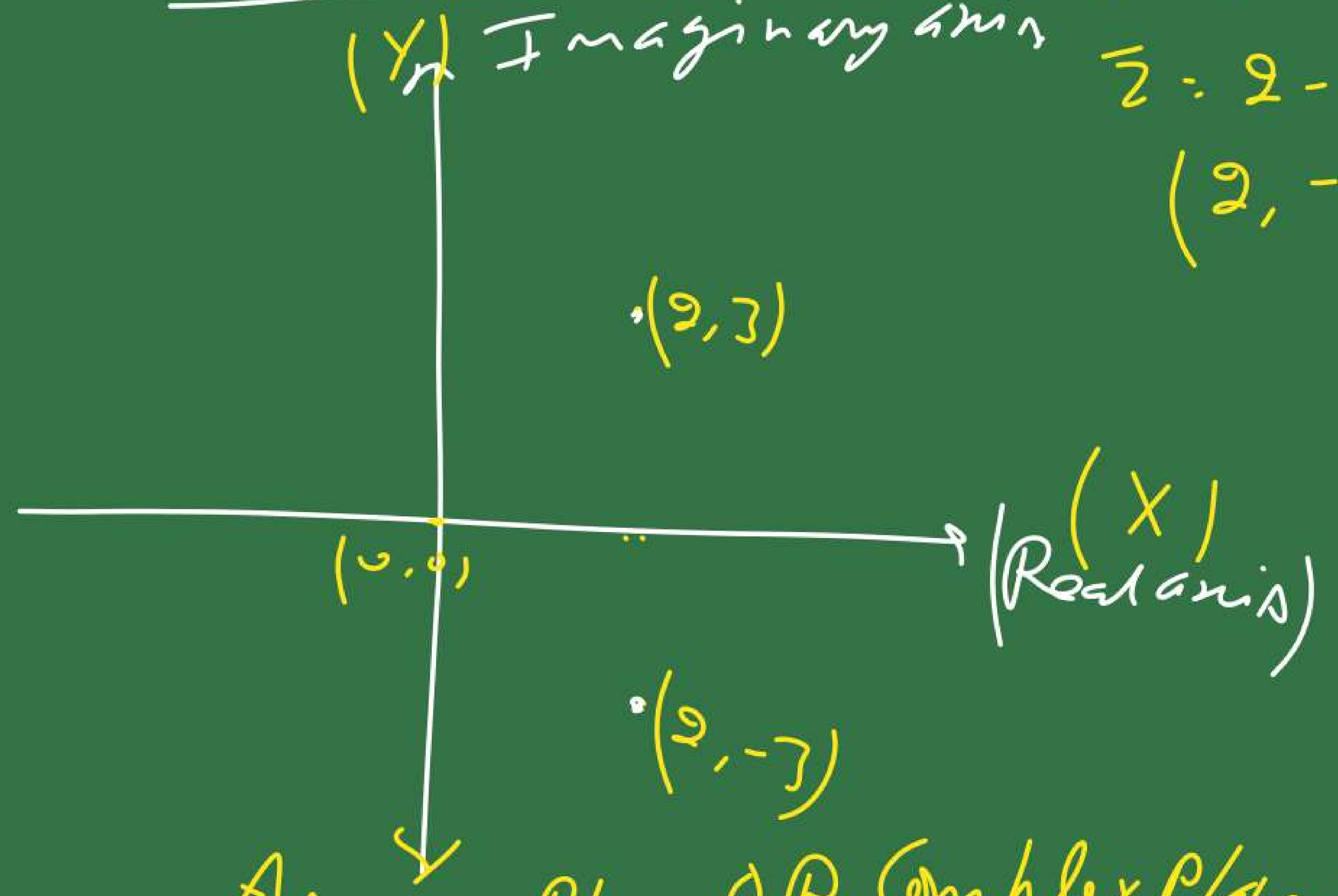
(i) Conjugate:

$z = 2 + 3i$

$\uparrow x \quad \uparrow y$

$\bar{z} = 2 - 3i$

$(2, -3)$

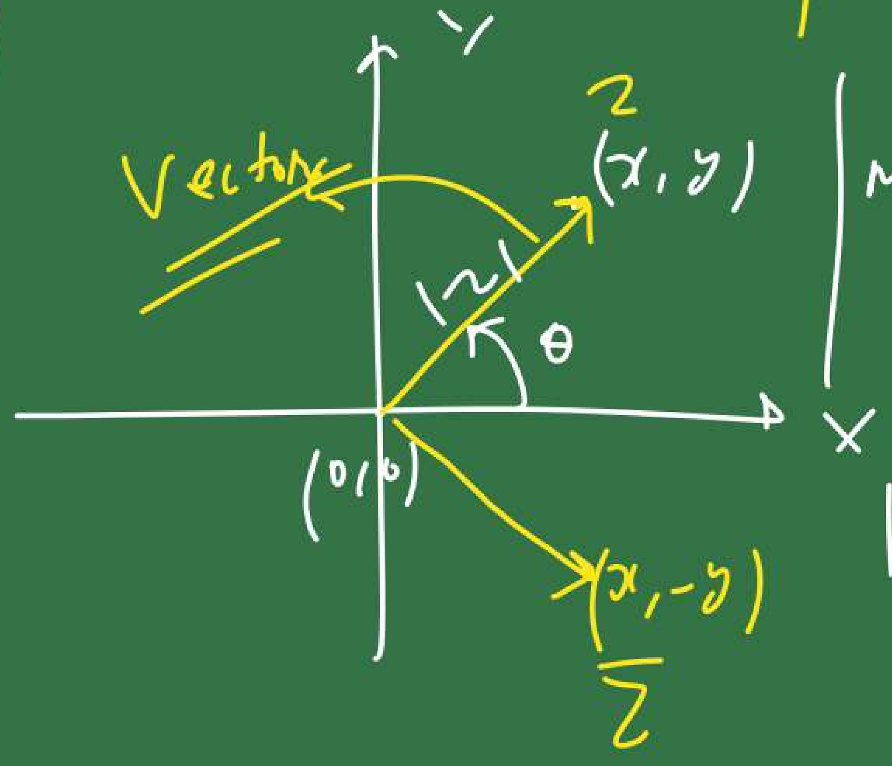


Argand Plane OR Complex Plane OR Gaussian Plane

$z = x + iy = (x, y)$

$\bar{z} = x - iy$

$(x, -y)$



magnitude $= |z|$

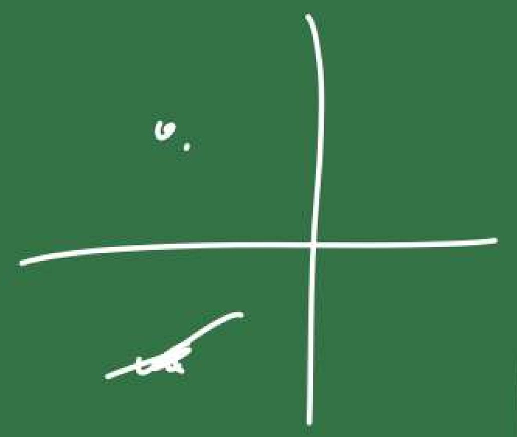
$|z| = \sqrt{x^2 + y^2}$

Note:- Conjugate of a Complex No is mirror image of z in Real axis.

let $z = -2+3i$

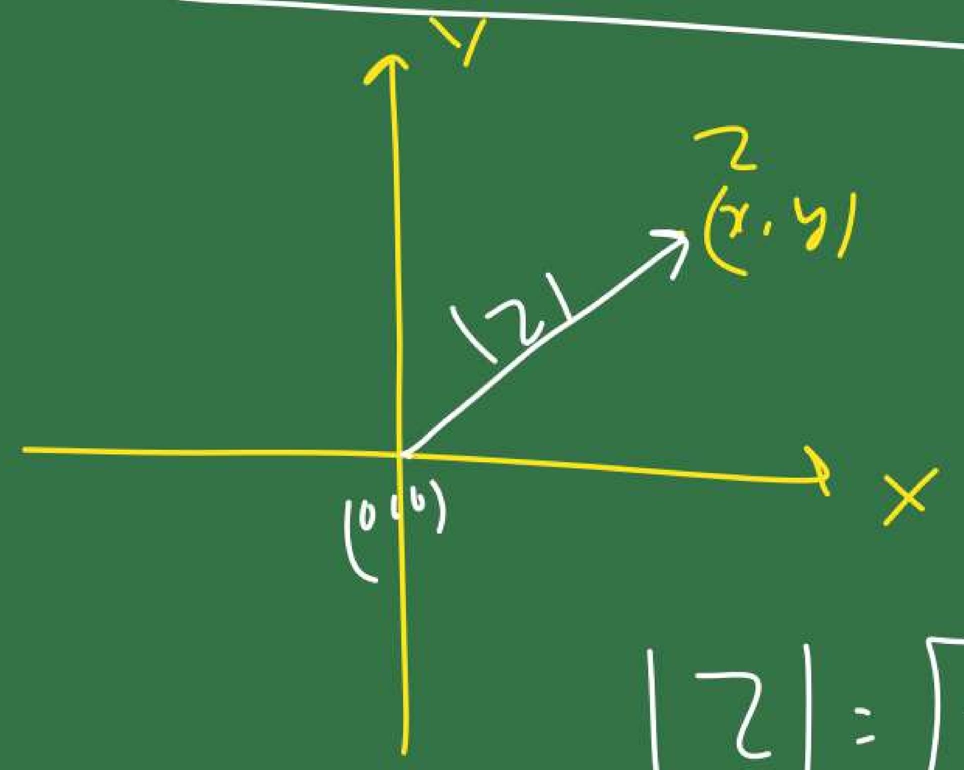
$= (-2, 3)$

IInd Quadrant



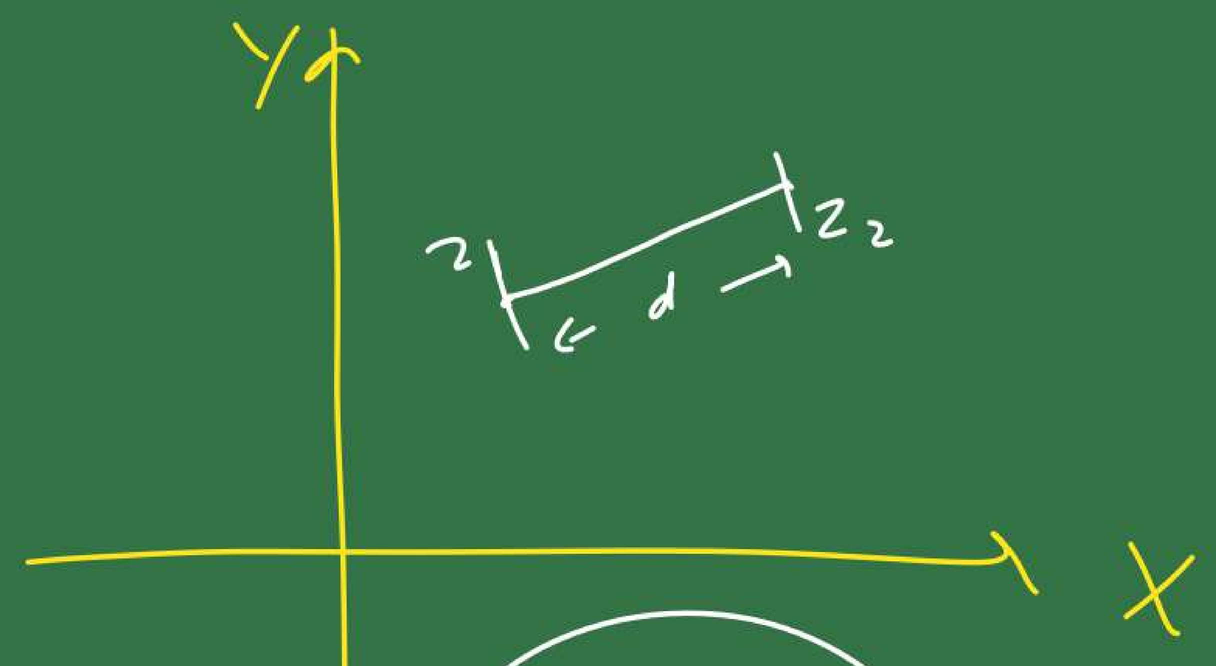
Conjugate is in 3rd Quadrant.

⇒ ② Modulus of Complex No:-



$$|z| = \sqrt{x^2 + y^2}$$

distance b/w two points in Argand Plane:



$d = |z_1 - z_2|$

for ex: $z_1 = -1+3i \;|\; z_2 = -2-4i$

distance b/w z_1 & z_2 is

$= |z_1 - z_2|$

$= |-1+3i-(-2-4i)|$

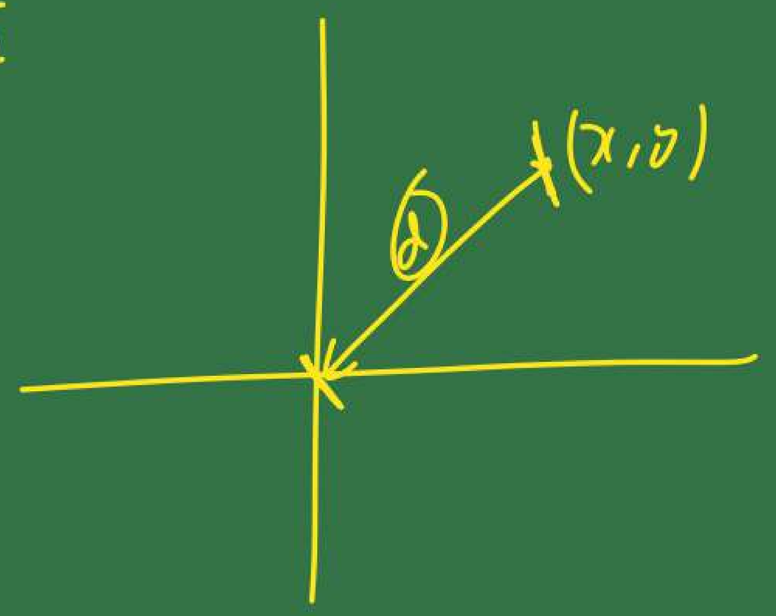
$= |-1+3i+2+4i|$

$= |1+7i| = \sqrt{50}$ ✓

$\therefore \quad |z| = |z - 0|$

origin

Generic: $|z| = \sqrt{x^2+y^2}$



Generic: (i) $|z+3| = |z-(-3)|$

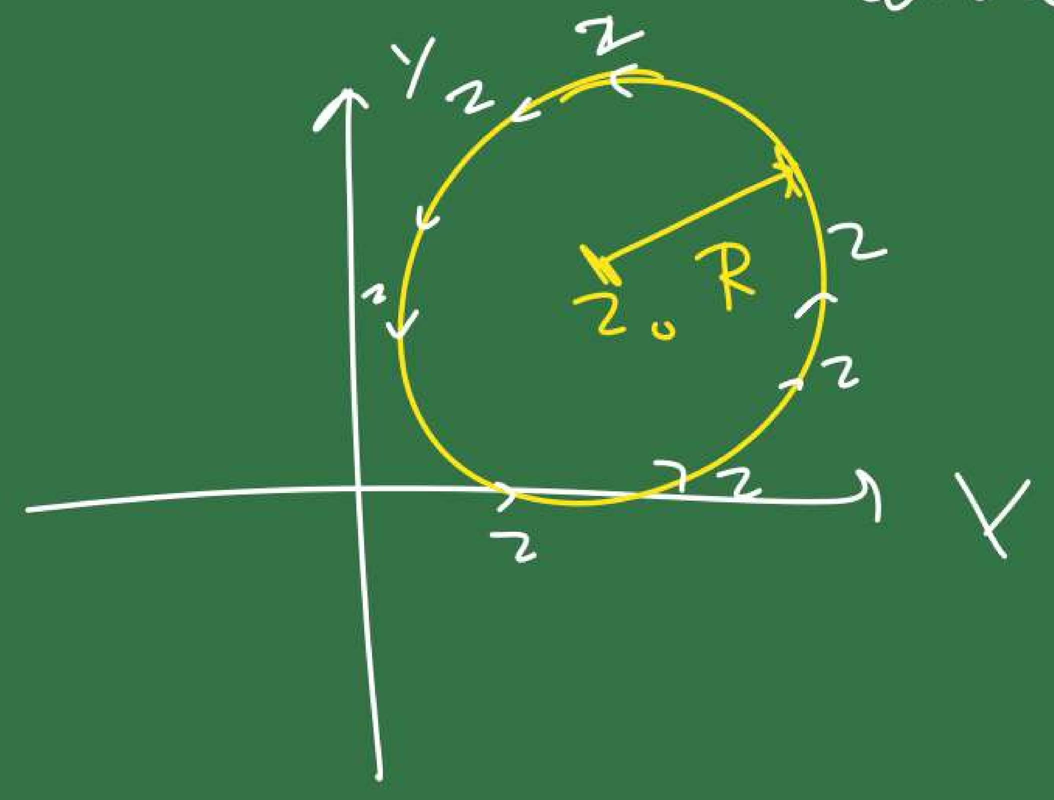
distance of z from $(-3, 0)$.

(ii) $|z-2+4i| = |z-(2-4i)|$ = distance of z from $(2, -4)$

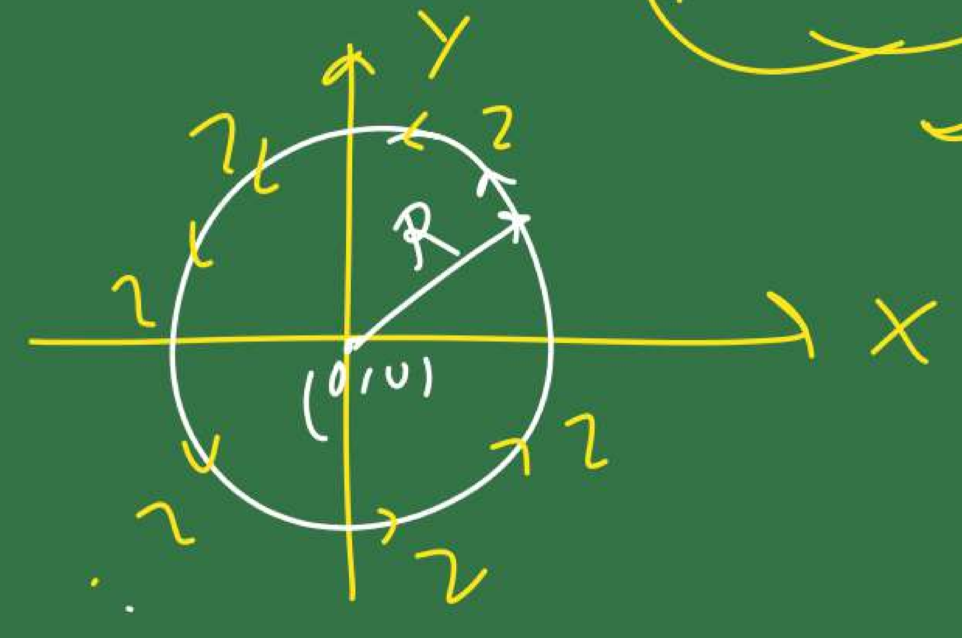
(iii) eqⁿ of Circle : $|z - z_0| = R$

where z_0 = Center of Circle .

R = Radius of Circle .



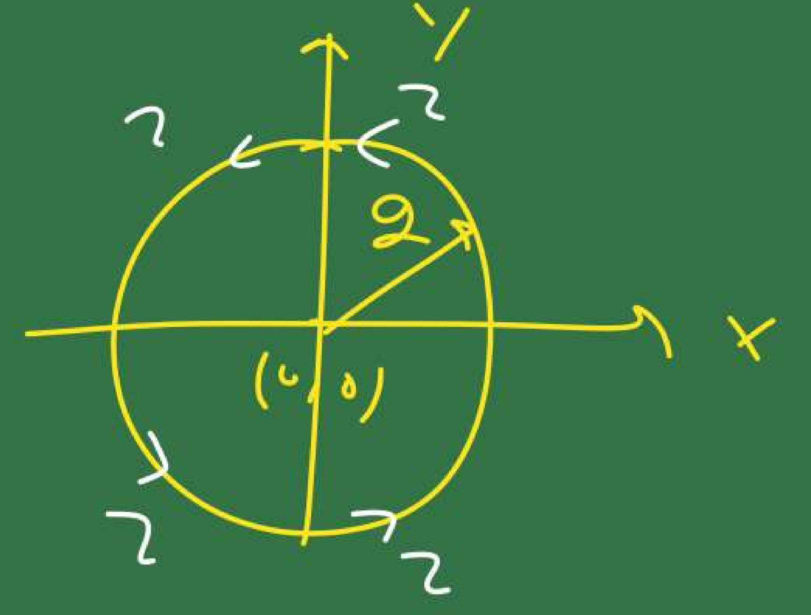
Let $z_0 = 0 \Rightarrow |z - 0| = R \Rightarrow |z| = R$



Point: (i) $|z| = 2$

↓

Circle.



Point: (ii) $|z+5| \le 4$ then minimum value of $|z|$?

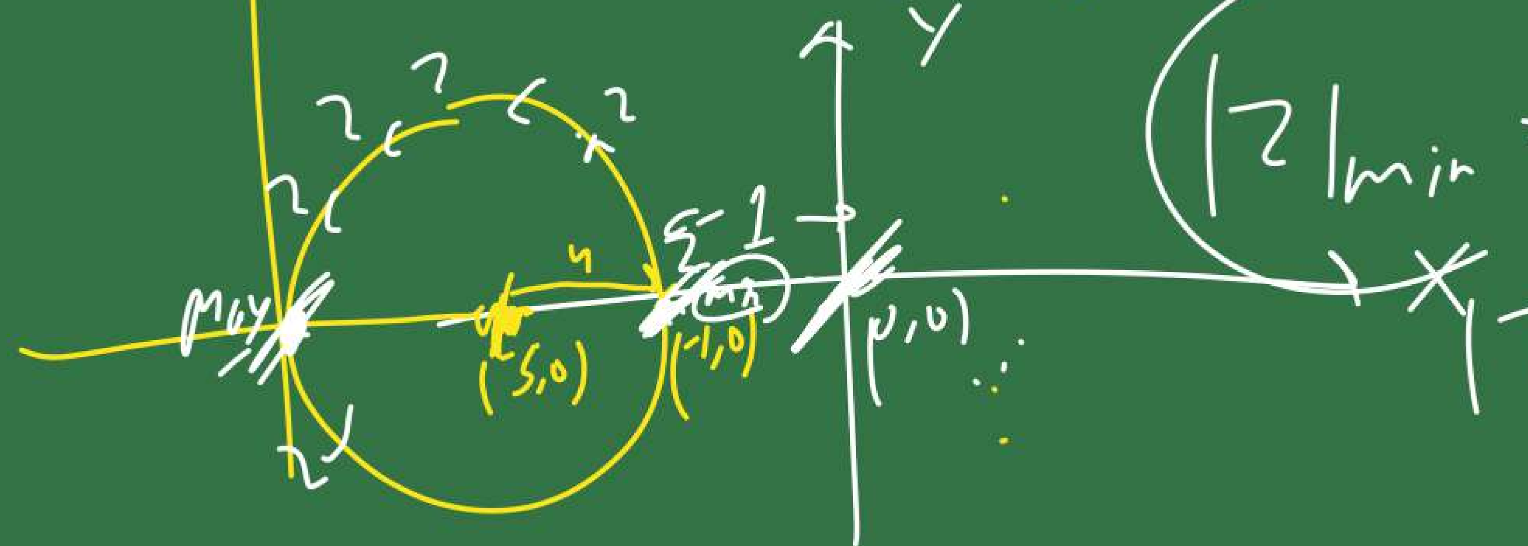
Sol: $|z+5| \le 4$

$|z-(-5)| \le 4$ Radius 4

↓

Center $(-5,0)$

$|z-0|$

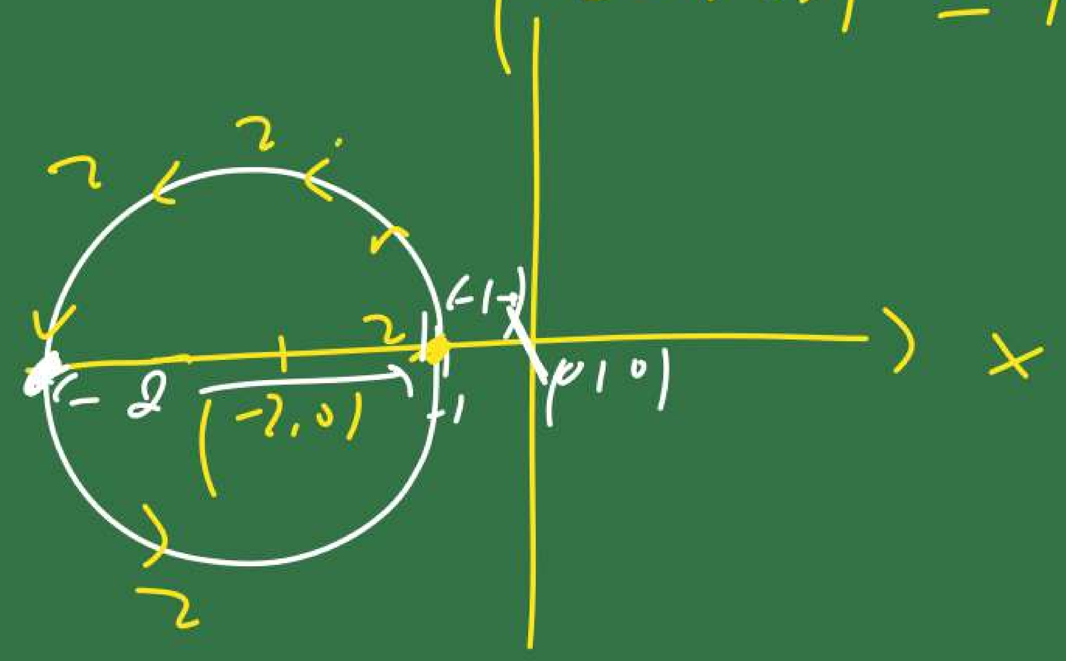


$|z|_{min} = 1$

$|z|_{max} = 9$

(ii) $|z+2| \le 1$ $\quad |z|_{max}$ & $|z|_{min}$?

$|z-(-2)| \le 1$ $\quad |z-0|$ $\quad$ 3 $\quad |z-0|$ $\quad$ 1



Conclusion:- (i) $|z - z_0| = R$ equation of circle.

$|z| = R \rightarrow$ equation of circle center $(0,0)$ Radius R.

(ii) $\boxed{z\bar{z} + a\bar{z} + \bar{a}z + b = 0}$ eqn of circle.

constant

Center $= (-a) =$ coefficient of $\bar{z}$

Radius $= \sqrt{|a|^2 - b}$

for ex: (i) $z\bar{z} + (2+3i)\bar{z} + (2-3i)z + 3 = 0$

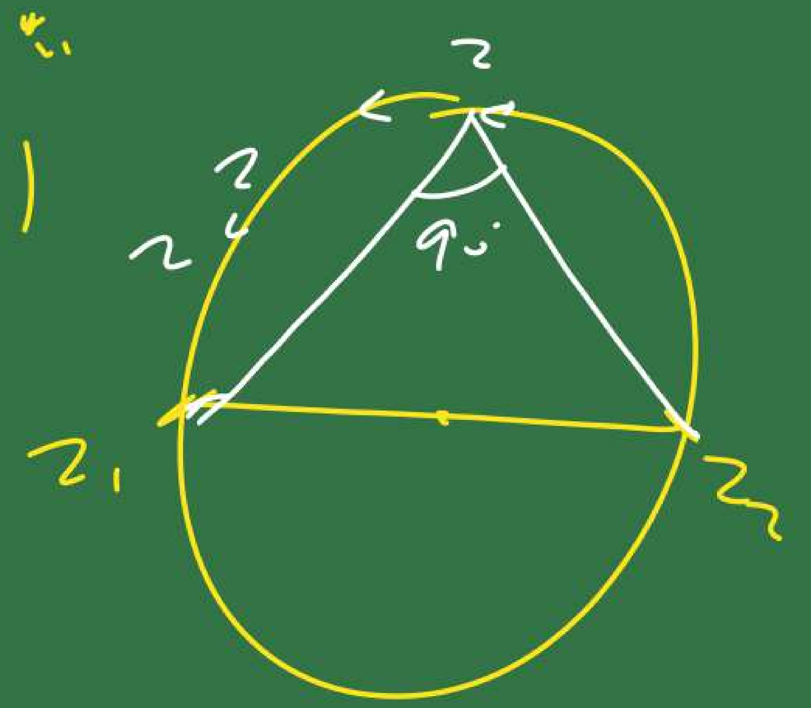
Center
$-(2+3i)$
$= (-2,-3)$

Radius
$= \sqrt{13-3}$
$= \sqrt{10}$

(iii) $|z - z_1| = k\,|z - z_2|$ where $k \neq 1$.

It is also representing equation of circle.

(iv)



$$\left(|z - z_1|\right)^2 + \left(|z - z_2|\right)^2 = |z_1 - z_2|^2$$

representing a circle.

z_1 & z_2 are end points of diameter.

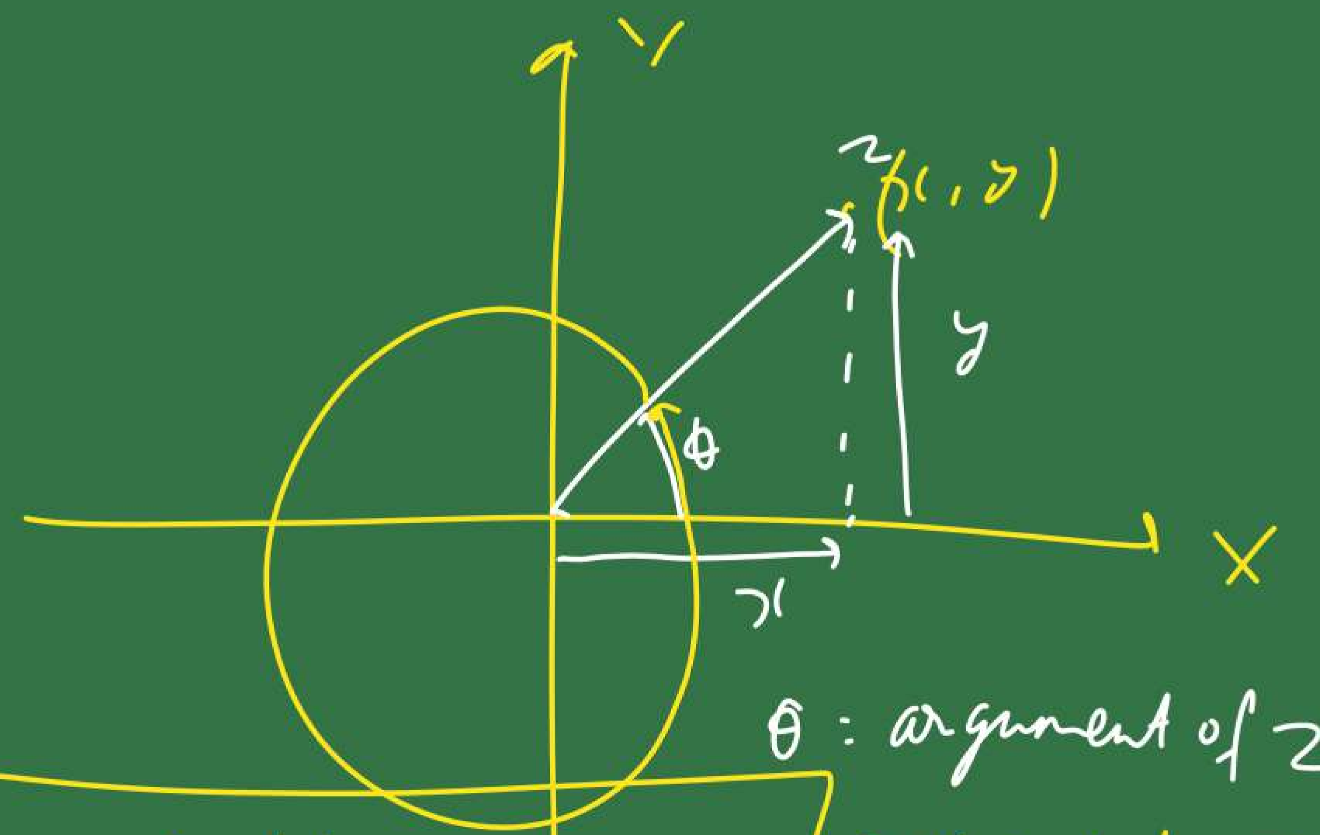
⇒ Polar form of Complex No:

$z = x + iy$

$z = r(\cos\theta + i\sin\theta)$

$$\boxed{\theta = \arg(z)}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$$



θ : argument of z

: angle made by z with +ve x-axis in Anticlockwise direction.

for every Complex No we can find argument by adding 2π OR subtracting 2π for ex: $z = 1 + i$

$\arg(z) = \theta = \tan^{-1}\left(\frac{1}{1}\right)$

$= \pi/4$

Possible argumen $\Rightarrow$ $2\pi + \pi/4$, $\frac{\pi}{4} - 2\pi$, $4\pi + \pi/4$

, $4\pi - \pi/4$

Principal Argument:

$-\pi < \theta \le \pi$

Algorithm to find Polar form:

Step 1: Write the Complex No. in Standard form.

Step 2: find r, $r = \sqrt{x^2+y^2} = |z|$

Step 3: find $\theta = \tan^{-1}\left(\left|\frac{y}{x}\right|\right)$

Step 4: Check Quadrant in which z is laying

	arg(z)
1st Quadrant.	θ
2nd	$\pi - \theta$
3rd	$-(\pi - \theta)$
4th	$-\theta$

Step 5: Put value of r & θ in Polar form.

$$z = r(\cos\theta + i\sin\theta)$$

Question: $z = -1 + \sqrt{3}i$ $(-1, \sqrt{3})$

$$r = |z| = \sqrt{1+3}$$
$$= \sqrt{4} = 2$$

$$\theta = \tan^{-1}\left(\left|\frac{\sqrt{3}}{-1}\right|\right)$$
$$= \tan^{-1}(\sqrt{3})$$
$$= \frac{\pi}{3}$$

$\Rightarrow$ z is in 2nd Quadrant

$$\pi - \frac{\pi}{3} = \frac{2\pi}{3}.$$

$$z = r(\cos\theta + i\sin\theta)$$

$$\boxed{z = 2\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)}$$

(ii) Euler form :-

$$\boxed{\cos\theta + i\sin\theta = e^{i\theta}}$$

$z = r(\cos\theta + i\sin\theta)$

$$\boxed{z = r e^{i\theta}}$$

⇒ De-Moivre's Theorem :- $(\cos\theta + i\sin\theta)^n$

$= \cos(n\theta) + i\sin(n\theta)$.

Note :- (i) $(\cos\theta + i\sin\phi)^n \neq (\cos(n\theta) + i\sin(n\phi))$

(ii) $(\sin\theta + i\cos\theta)^n \neq \sin(n\theta) + i\cos(n\theta)$

Very Imp Trick! (M.S.T): Mohit Sir Trick!

(1) $\left| z \pm \frac{b}{z} \right| = a$

$|z|_{max} = \frac{a + \sqrt{a^2 + 4b}}{2}$

$|z|_{min} = \frac{-a + \sqrt{a^2 + 4b}}{2}$.

(2) $\frac{\sqrt{3}+i}{\sqrt{3}-i} = (i)^{2/3}$ | $\frac{\sqrt{3}-i}{\sqrt{3}+i} = (i)^{-2/3}$ | $\frac{1+i}{\sqrt{2}} = (i)^{1/2}$

$\frac{1-i}{\sqrt{2}} = (-i)^{1/2}$

(3) Square root of $z = x+iy$:

$$\sqrt{x+iy} = \pm\left(\sqrt{\frac{|z|+x}{2}} + i\sqrt{\frac{|z|-x}{2}}\right)$$

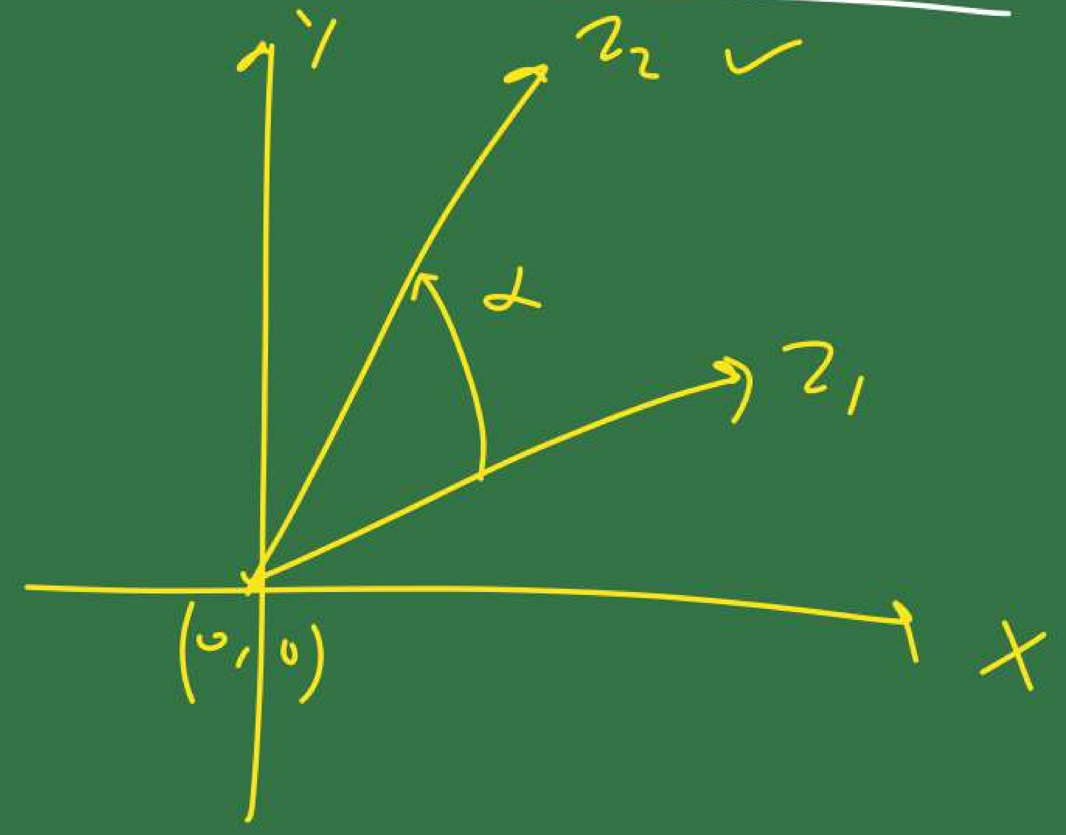
$$\sqrt{x-iy} = \pm\left(\sqrt{\frac{|z|+x}{2}} - i\sqrt{\frac{|z|-x}{2}}\right)$$

Mohit Sir Trick : (MST)

$\sqrt{x+iy} = \pm(\alpha+i\beta)$ then $\sqrt{-x+iy} = \pm(\beta+i\alpha)$

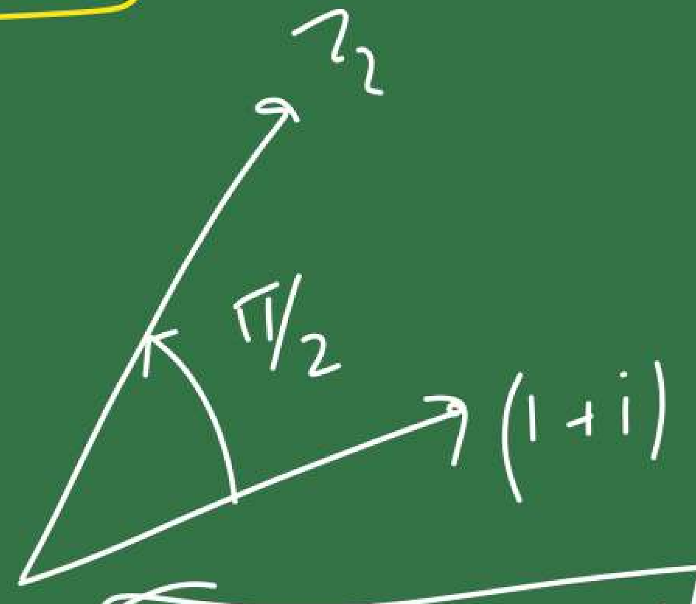
$$\sqrt{-x-iy} = \pm(\beta - i\alpha)$$

Rotation Theorem:



$$\boxed{z_2 = z_1 e^{i(\alpha)}}$$

Given:



$$\boxed{z_2 = (1+i)\, e^{i(\pi/2)}}$$

$z_2 = (1+i)\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

$= (1+i)(0+i)$

$= i(1+i)$

$= i + i^2$

$z_2 = i - 1$

Important Note:

(i) If we multiply any Complex No. (z) with (i), then it means we rotated z by $\frac{\pi}{2}$ in anti-clockwise sense.

(ii) If we multiply any Complex No. (z) with ($-i$) then it means, we rotated z by $\pi/2$ (clockwise)

⇒ Cube root of Unity:

$$z^3 = 1$$

$$z = (1)^{1/3}$$

$$z^3 = 1$$

$$z^3 - 1 = 0$$

(x) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$(z-1)(z^2+z+1) = 0$$

$z_1 = 1$ $\qquad z^2 + z + 1 = 0$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$z = \frac{-1 \pm \sqrt{3}\,i}{2}$$

$z_2 = \frac{-1}{2} + \frac{i\sqrt{3}}{2}$ $\qquad z_3 = \frac{-1}{2} - \frac{i\sqrt{3}}{2}$

ω $\qquad \omega^2$

Important Notes:

(i) $z = (1)^{1/3}$

$z = 1, \frac{-1}{2} + \frac{i\sqrt{3}}{2}, \frac{-1}{2} - \frac{i\sqrt{3}}{2}$

$\boxed{z = 1, \omega, \omega^2}$

$z = 1, e^{i\left(\frac{2\pi}{3}\right)}, e^{i\left(\frac{4\pi}{3}\right)}$

$= 1, \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right), \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)$

(ii) $z^3 = 1$

$z^3 - 1 = (z-1)(z-\omega)(z-\omega^2)$

Replace z with x.

$x^3 - 1 = (x-1)(x-\omega)(x-\omega^2)$.

(iii) $z^3 = -1 \rightarrow z = -1, -\omega, -\omega^2$

$z^3 + 1 = 0$

$z^3 + 1 = (z+1)(z+\omega)(z+\omega^2)$.

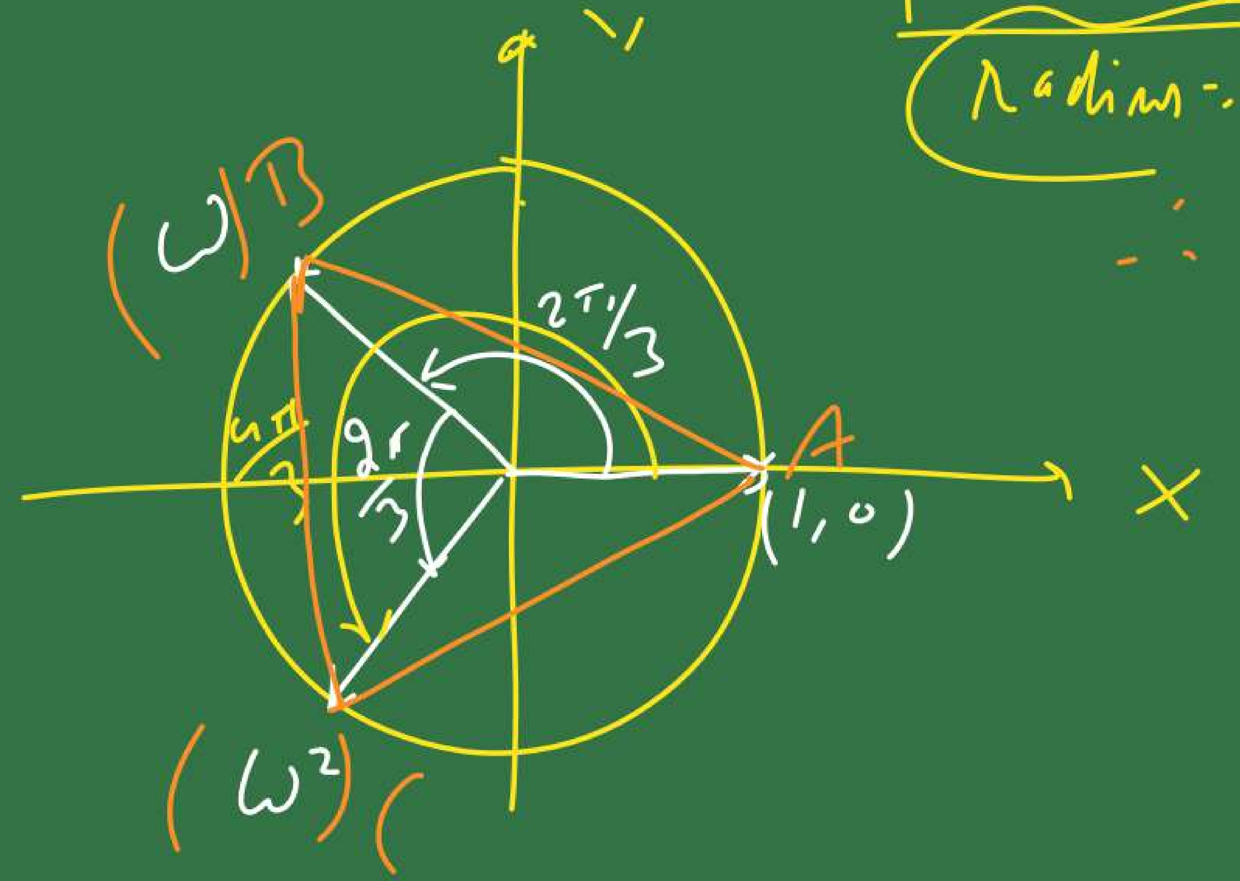
(iv) Sum of cube roots of unity $= 0$

$\boxed{1 + \omega + \omega^2 = 0.}$

(v) $(\omega)^3 = 1$ (bcz $e^{i(2\pi)} = 1$)

(vi) Cube root of unity divides the $\boxed{\text{unit circle}}$ in 3 equal parts.

Radius = 1.



∴ ABC is an equilateral Triangle.

(7) $z^2+z+1=0$

$z=\omega, \omega^2$

$z^2-z+1=0$

$z=-\omega, -\omega^2$

⇒ Argument of Complex No :-

All these properties are Not Valid for Principal Argument.

① $\arg(\bar{z}) = -\arg(z)$

② $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

③ $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

④ $\arg(z^n) = n\arg(z)$

⑤ If $\arg(z_1) + \arg(z_2) = 0$ then z_1 & z_2 are Conjugate of each other.

$\therefore z_1 = \bar{z}_2$

$\therefore \arg(z_1) = -\arg(z_2)$

$\boxed{\arg(z_1) = \arg(\bar{z}_2)}$

(1) $\arg(\omega) = \frac{2\pi}{3}$

$\arg(\omega^2) = \frac{4\pi}{3}$.

Important Notes:-

(i) Area of triangle by z, iz & $z+iz$ is

$= \frac{1}{2}|z|^2$

(2) Area of triangle by z, ωz & $z+\omega z$ is

$= \frac{\sqrt{3}}{4}|z|^2$

(3) If $|z| = 1$ then assume, $z = 1, \omega, \omega^2$

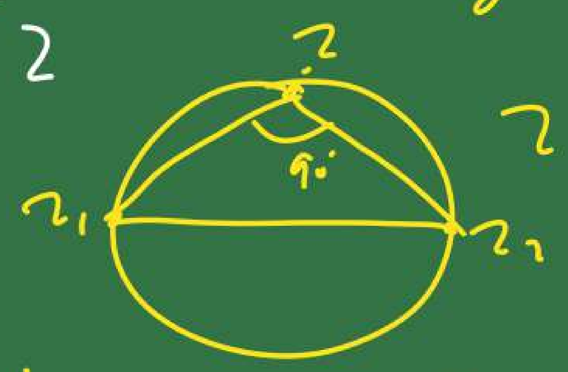
(4) $z^2 = n(\bar{z})$

Total roots $= 4 = (0,0)$ & 3 Non-zero

Non zero roots $= 3$.

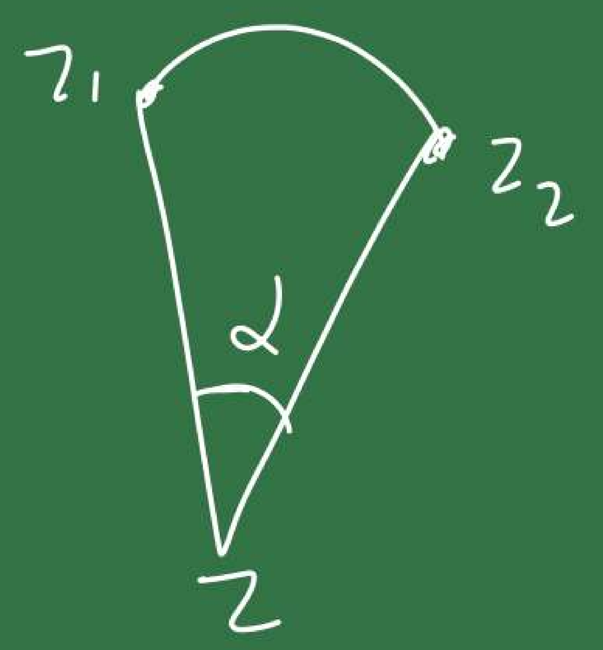
$n(1), n(\omega), n(\omega^2)$ OR $n(-1), n(-\omega), n(-\omega^2)$

(5) $\arg\left(\dfrac{z-z_1}{z-z_2}\right) = \pm\dfrac{\pi}{2}$ → It represent a circle & z_1 & z_2 are end points of diameter.



(6) $\arg\left(\dfrac{z-z_1}{z-z_2}\right) = \alpha$ → Represent a Sector with angle α.

↑ any angle.



complex number is given by $z = \dfrac{1+2i}{1-(1-i)^2}$

$-2i$

08. What is the modulus of z? [2019-I]

(a) 4 (b) 2

(c) 1 (d) $\dfrac{1}{2}$

09. What is the principal argument of z? [2019-I]

(a) 0 (b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{2}$ (d) π

$z = \dfrac{1+2i}{1-(-2i)} = \dfrac{1+2i}{1+2i} = 1$

$\boxed{z = 1+0i}$

$z = x+iy$

$\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$

$|z| = 1$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

NDA (21
2022)

(i) $z = x + 0i$

$\theta = \tan^{-1}\left(\frac{0}{x}\right)$

$= \pi$

$= 2\pi$

$= 3\pi$

$n\pi$

$(2n+1)\frac{\pi}{2}$ $z = 0 + yi$

$\theta = \tan^{-1}\left(\frac{y}{0}\right) = \frac{\pi}{2}$

$= \frac{3\pi}{2}, \frac{5\pi}{2}$

. Which one of the following is correct in respect of the cube roots of unity? *[2018-II]*

(a) They are collinear

(b) They lie on a circle of radius $\sqrt{3}$

(c) They form an equilateral triangle

(d) None of the above

The number of non-zero integral solutions of the equation $|1-2i|^x = 5^x$ is [2018-I]

(a) Zero (No solution) (b) One

(c) Two (d) Three

$$(|1-2i|)^x = (5)^x$$

$$\left(\sqrt{(1)^2+(-2)^2}\right)^x = (5)^x$$

$$(\sqrt{5})^x = (5)^x$$

$$(5)^{x/2} = 5^x$$

$$\frac{5^x}{5^{x/2}} = 1$$

$$5^{x-\frac{x}{2}} = (5)^0$$

$$5^{x/2} = (5)^0$$

$$\frac{x}{2} = 0$$

$$x = 0$$

If α and β are different complex numbers with $|\alpha| = 1$, then what is $\left|\frac{\alpha-\beta}{1-\alpha\beta}\right|$ equal to? *[2018-I]*

(a) $|\beta|$ (b) 2
(c) 1 (d) 0

$|\alpha| = 1$

let $\alpha = 1$

$$\left|\frac{\cancel{1-\beta}}{\cancel{1-\beta}}\right| = 1$$

The number of roots of the equation $z^2 = 2\bar{z}$ is [2017-I]

(a) 2 (b) 3

(c) 4 (d) zero

If $|z+5| \le 3$, then the maximum value of $|z+1|$ is *[2017-I]*

(a) 0 (b) 4

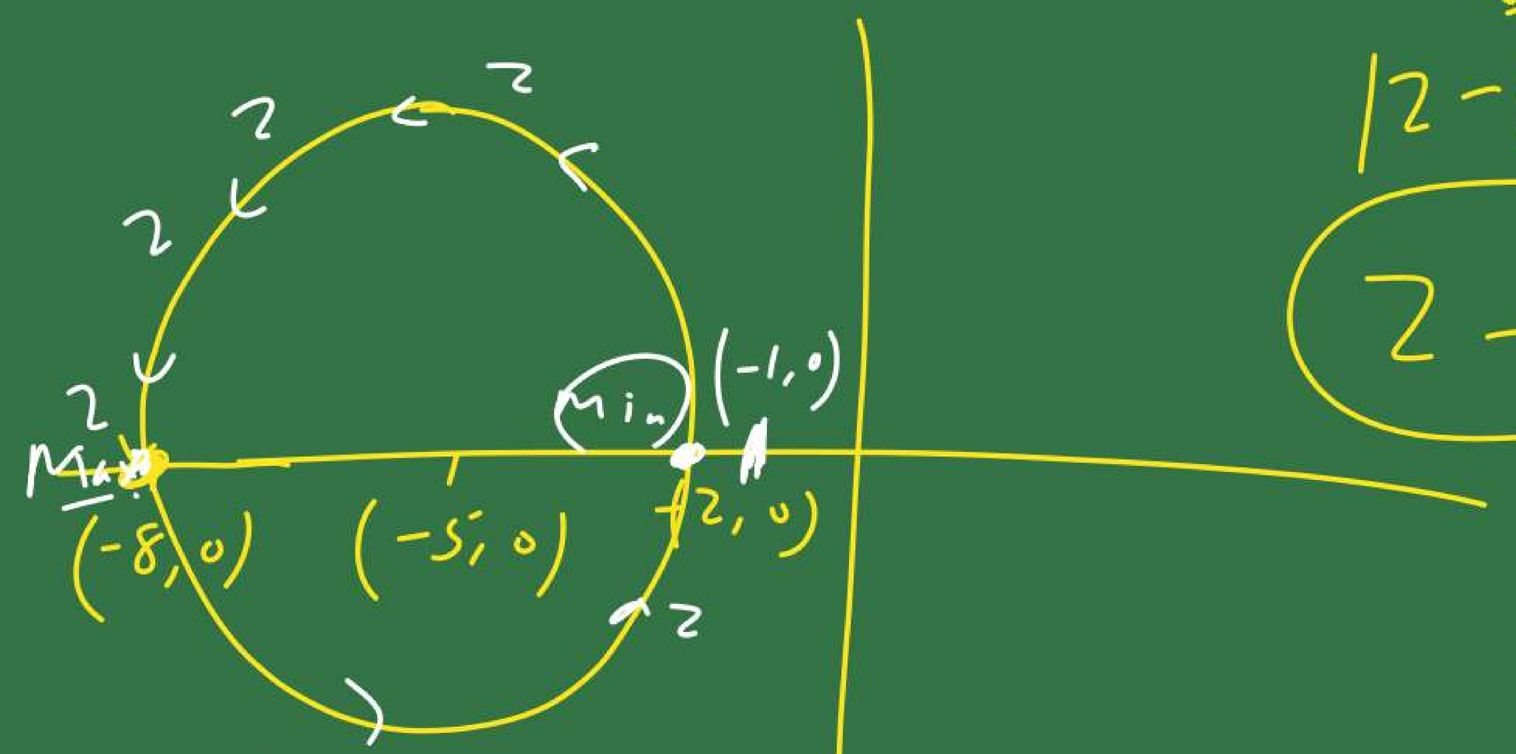
(c) 6 (d) 10

Ready

PYQ

$|z-(-1)|$

$z \to (-1, 0)$



Max = 7

Min = 1

Let z be a complex number satisfying [2016-I]

$$\left|\frac{z-4}{z-8}\right| = 1 \text{ and } \left|\frac{z}{z-2}\right| = \frac{3}{2}$$

80. What is $|z|$ equal to?

(a) 6 (b) 12

(c) 18 (d) 36

$6 \quad 6-6=0$

81. What is $\left|\frac{z-6}{z+6}\right|$ equal to?

(a) 3 (b) 2

(c) 1 (d) 0

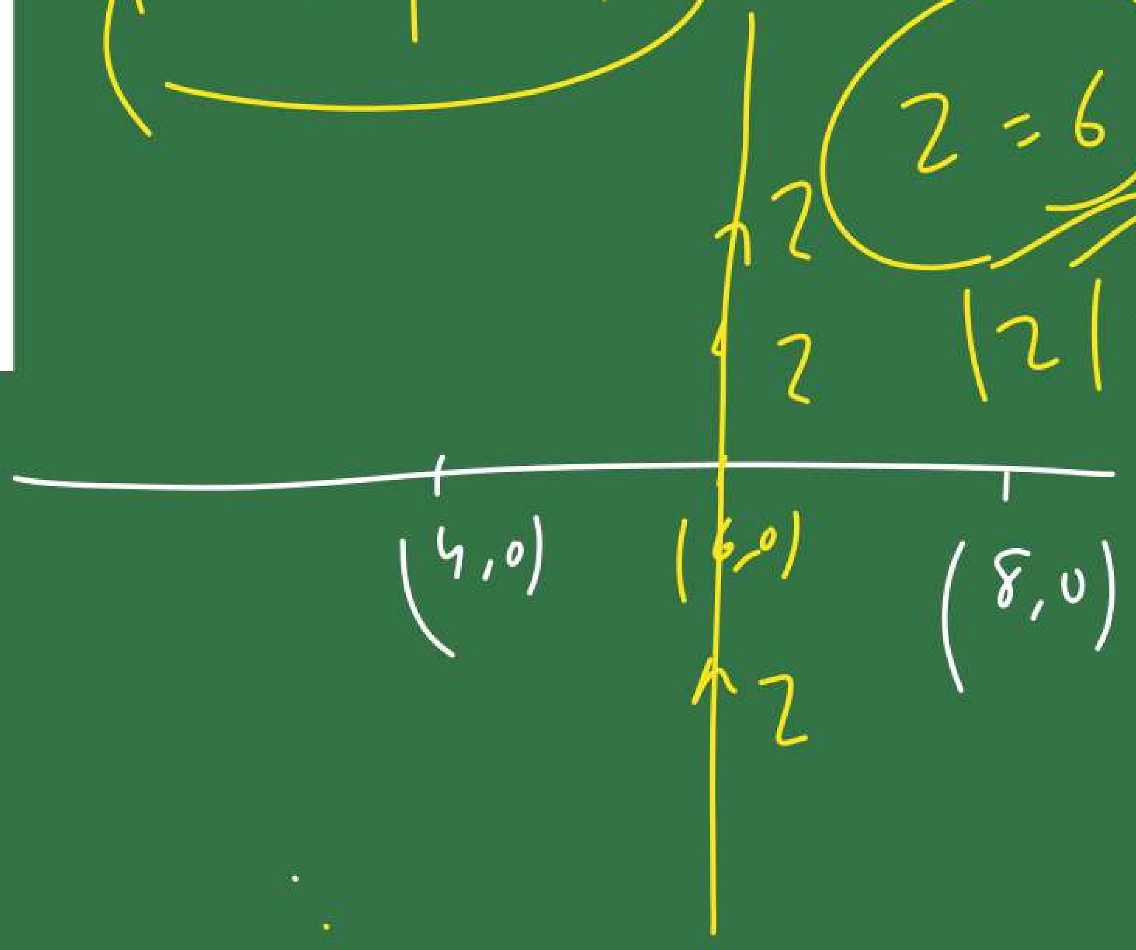
$$\frac{|z-4|}{|z-8|} = 1$$

$$|z-4| = |z-8|$$

$$|z| = \frac{3}{2}|z-2|$$

$$z = 6$$

$$|z| = 6$$



$$d = |z_1 - z_2|$$

If $\text{Re}\left(\frac{z-1}{z+1}\right) = 0$, where $z = x + iy$ is a complex number, then which one of the following is correct? *[2016-II]*

(a) $z = 1 + i$

(b) $|z| = 2$

(c) $z = 1 - i$

(d) $|z| = 1$

काल्पनिक संख्या !

let $z = x + iy$

$$\frac{x+iy-1}{x+iy+1} \times \frac{(x+1)-iy}{(x+1)-iy} = \text{Real part} = 0$$

$z = ?$

Purely Imaginary :

$\bar{z} = -z$

$$\overline{\left(\frac{z-1}{z+1}\right)} = -\left(\frac{z-1}{z+1}\right)$$

$$\left(\frac{\bar{z}-1}{\bar{z}+1}\right) \times \frac{-(z-1)}{(z+1)}$$

$$\bar{z}z + \bar{z} - z - 1 = -(\bar{z}z - \bar{z} + z - 1)$$

$$|z|^2 + \cancel{\bar{z}} - \cancel{z} - 1 = -|z|^2 + \cancel{\bar{z}} - \cancel{z} + 1$$

$\cancel{2}|z|^2 = \cancel{2}$

$|z|^2 = 1$

$|z| = 1$

If $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{107}+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^{107}$, then what is the imaginary part of z equal to? *[2016-II]*

(a) 0
(b) $\frac{1}{2}$
(c) $\frac{\sqrt{3}}{2}$
(d) 1

$$\bar{z}=\left(\overline{\frac{\sqrt{3}}{2}+\frac{i}{2}}\right)^{107}+\left(\overline{\frac{\sqrt{3}}{2}-\frac{i}{2}}\right)^{107}$$

$$=\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^{107}+\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{107}$$

$$\bar{z}=z$$

Purely Real.

Let z_1, z_2 and z_3 be non-zero complex numbers satisfying $z^2 = i\bar{z}$, where $i = \sqrt{-1}$. *[2016-I]*

78. What is $z_1 + z_2 + z_3$ equal to?
    (a) i (b) $-i$
    (c) 0 (d) 1

79. Consider the following statements:
    1. $z_1 z_2 z_3$ is purely imaginary.
    2. $z_1 z_2 + z_2 z_3 + z_3 z_1$ is purely real.

    Which of the above statements is/are correct?
    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) Neither 1 nor 2

$z^2 = i\bar{z}$ → 4 roots

$z^2 = n\bar{z}$

$(0,0)$ $(i(1))$, $(i\omega)$, $(i\omega^2)$

OR

$(0,0)$ $(i(-1))$, $(i(-\omega))$, $(i(-\omega^2))$

$z_1 = i$ | $z_2 = i\omega$ | $z_3 = i\omega^2$

(i) $i(1+\omega+\omega^2) = i \times 0 = 0$

→ $z_1 z_2 z_3 = i^3(\omega^3) = i^3 \times 1 = -i$

→ $i^2(\omega + \omega^{4} + \omega^2)$

$i^2(1+\omega+\omega^2) = 0$

$(x^3 - 1)$ can be factorised as *[2015-I]*

(a) $(x-1)(x-\omega)(x+\omega^2)$

(b) $(x-1)(x-\omega)(x-\omega^2)$

(c) $(x-1)(x+\omega)(x+\omega^2)$

(d) $(x-1)(x+\omega)(x-\omega^2)$

What is [2015-I]

$$\left[\frac{\sin\frac{\pi}{6}+i\left(1-\cos\frac{\pi}{6}\right)}{\sin\frac{\pi}{6}-i\left(1-\cos\frac{\pi}{6}\right)}\right]^3$$

where $i=\sqrt{-1}$, equal to?

(a) 1 (b) –1

(c) i (d) $-i$

$1-\cos(\theta) = 2\sin^2\left(\frac{\pi}{12}\right)$, $\theta = \frac{\pi}{6}$

$$\boxed{\sin\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)}$$

$$\left(\frac{2\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)+i\,2\sin^2\left(\frac{\pi}{12}\right)}{2\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)-i\,2\sin^2\left(\frac{\pi}{12}\right)}\right)^3$$

$$\left(\frac{\cancel{2\sin\left(\frac{\pi}{12}\right)}\boxed{\left(\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)\right)}}{\cancel{2\sin\left(\frac{\pi}{12}\right)}\left(\cos\left(\frac{\pi}{12}\right)-i\sin\left(\frac{\pi}{12}\right)\right)}\right)^3$$

$$\left(\frac{e^{i\theta}}{e^{i(-\theta)}}\right)^3 = \left(e^{i2\theta}\right)^3 = e^{i6\theta} = e^{i6\times\frac{\pi}{12}} = e^{i(\pi/2)}$$

$$\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right) = i = (1)i$$

If 1, ω, ω^2 are the cube roots of unity, then the value of $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ is *[2015-I]*

(a) -1 (b) 0

(c) 1 (d) 2

$1+\omega+\omega^2=0$

$1+\omega^2=-\omega$

$\omega+\omega^2=-1$

$\omega^4=\omega^3\cdot\omega$

$=\omega$

$\omega^8=\omega^6\cdot\omega^2$

$=1\times\omega^2$

$(1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2)$

$-\omega$

$(1+\omega)(-\omega)$

$-(\omega+\omega^2)$

$-1\times-1=1$

$\times(1)=1\times1=1$

2. Let $z = x + iy$ Where x, y are real variables $i = \sqrt{-1}$. If $|2z-1| = |z-2|$, then the point z describes : *[2014-I]*

(a) A circle (b) An ellipse
(c) A hyperbola (d) A parabola

$2\left|z - \frac{1}{2}\right| = |z-2|$

$\frac{\left|z - \frac{1}{2}\right|}{|z-2|} = \frac{1}{2}$

$\left|z - \frac{1}{2}\right| = \frac{1}{2}|z-2|$

Circle